

Program-Aided Reasoners (*Better*) Know What They Know

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Abstract

Prior work shows that program-aided reasoning, in which large language models (LLMs) are combined with programs written in programming languages such as Python, can significantly improve accuracy on various reasoning tasks. However, while accuracy is essential, it is also important for such reasoners to “know what they know”, which can be quantified through the *calibration* of the model. In this paper, we compare the calibration of Program Aided Language Models (PAL) and text-based Chain-of-thought (CoT) prompting techniques over 5 datasets and 2 model types - LLaMA models and OpenAI models. Our results indicate that PAL leads to improved calibration in 75% of the instances. Our analysis uncovers that prompting styles that produce lesser diversity in generations also have more calibrated results, and thus we also experiment with inducing lower generation diversity using temperature scaling and find that for certain temperatures, PAL is not only more accurate but is also more calibrated than CoT. Overall, we demonstrate that, in the majority of cases, program-aided reasoners *better* know what they know than text-based counterparts.¹

1 Introduction

As language models (LMs) grow in size and capabilities, several works examine methods to improve their reasoning skills with different styles of prompting (Wei et al., 2022; Wang et al., 2022; Suzgun et al., 2022b; Zhou et al., 2022; Yao et al., 2023). One representative method, chain of thought (CoT) reasoning (Wei et al., 2022), takes inspiration from how humans approach problem-solving – by breaking down the problem into a sequence of natural language explanations before arriving at a final answer. Furthermore, prompts that enable problem-solving are not limited to natural

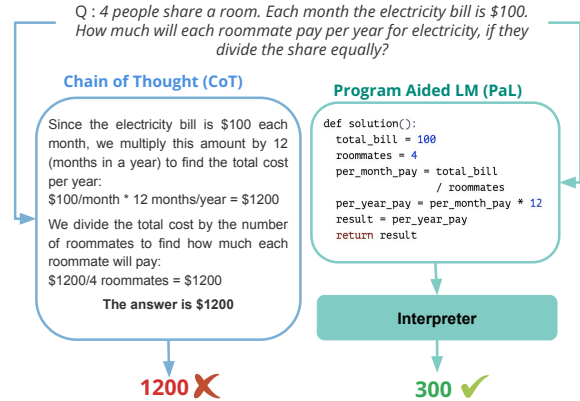


Figure 1: Comparisons of CoT and PAL outputs. CoT can sometimes generate the correct reasoning chain but fail to derive the correct answer as a final step, PAL fixes this issue by executing generated code to arrive at a deterministic answer.

language; program-aided language models (PAL); Gao et al. (2022) have demonstrated the efficacy of using code (such as Python programs) as a means of improving the model’s reasoning, surpassing the accuracy of conventional chain-of-thought style prompts in some tasks (Madaan et al., 2022; Lyu et al., 2023; Zhang et al., 2023a,b). An illustration of both methods is shown in Figure 1.

Currently, most works proposing such methods have been primarily focused on improving accuracy. However, for real-world applications, another highly desirable feature of ML systems is that they should be able to provide *reliable confidence estimates*. Accurate estimates of model confidence are helpful for many applications, including allowing the model to refrain from providing an answer when uncertain, asking for human intervention in uncertain cases, or providing confidence estimates to a downstream model that consumes the outputs. The reliability is measured through *calibration*, how a model’s confidence in its predictions aligns accurately with real outcomes (Guo et al., 2017a; Jiang et al., 2020; Zhao et al., 2021).

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¹Code and data are available at <https://github.com/mathuryash5/code-calibrates>.

In sum, the previous research has shown, as eloquently stated by Kadavath et al. (2022) “language models (mostly) know what they know” — LLMs are reasonably well calibrated, although some imperfections remain.

In this work, we examine the effect of program-aided reasoning on calibration. We consider 5 datasets that cover different reasoning tasks and evaluate the performance of both PAL and CoT style prompting for OpenAI models (OpenAI, 2023) and LLaMA models (Touvron et al., 2023) with respect to accuracy and calibration. We primarily explore three main research questions :

- **RQ 1:** Does program-aided reasoning result in significantly different calibration than text-based CoT?
- **RQ 2:** Are the observed trends different across OpenAI models and LLaMA models?
- **RQ 3:** Does the consistency of LLM generations affect calibration? We examine this by measuring generation diversity and answer space entropy.

Our results show that program-aided reasoners know what they know *even better* than standard text-based reasoners with CoT. In particular, on OpenAI models, PAL exhibits not only superior accuracy, but also a consistent enhancement in calibration, of about 50%, over CoT. Interestingly, the consistent improvement of calibration is not observed in LLaMA models, but we find that by adjusting the temperature of sampling (similar to a widely used method of Platt scaling (Platt et al., 1999)), PAL improves with respect to both accuracy and calibration. We also conduct a detailed analysis of these observations, and find a correlation between the similarity of the generated chains-of-thoughts or programs and calibration which might help in explaining these trends.

2 Preliminaries and Mathematical Formulation

2.1 Measuring Calibration

Calibration refers to the alignment between the predicted probability estimates of a model and their actual correctness or accuracy (Guo et al., 2017b). Formally a perfectly calibrated model can be expressed using the following equation, where X is the given input, Y is the true output, the model’s

output is \hat{Y} and $P_N(\hat{Y} | X) = p$ is the probability, or “confidence”, over the model’s output.

$$P(\hat{Y} = Y | P_N(\hat{Y} | X) = p) = p, \forall p \in [0, 1] \quad (1)$$

In essence, Equation 1 conveys that if a perfectly calibrated model makes 100 predictions, and the confidence of each prediction is 0.6 then we expect the accuracy to be also 0.6. Nevertheless, the model may exhibit varying confidence levels for each sample. Therefore, it is imperative to calculate calibration across all confidence scores. In practice, we estimate this probability by dividing the predictions into M separate and equally sized interval buckets based on their confidence levels.

We use the expected calibration error (ECE), a common measure of (lack of) calibration which is a weighted average of the discrepancy between each bucket’s accuracy and confidence. It is given in Equation 2

Here B_m is the m -th bucket that contains samples whose probabilities of predictions fall in the interval $(\frac{m-1}{M}, \frac{m}{M}]$, where $\frac{|B_m|}{n}$ is B_m ’s size relative to all the samples. $\text{acc}(B_m)$ is the average accuracy of the samples in the m -th bucket, and $\text{conf}(B_m)$ is the corresponding average confidence of the samples falling in the m -th bucket.

$$\sum_{m=1}^M \frac{|B_m|}{n} |\text{acc}(B_m) - \text{conf}(B_m)| \quad (2)$$

Consider a setup where we have buckets with a step size of 0.1. All instances where a model assigns probabilities between 0.4 and 0.5 will be allocated to the bucket B_4 or the bucket encompassing probabilities between 0.4 and 0.5. We then calculate the average accuracy for the instances in these buckets along with the average probability/confidence. The absolute difference is multiplied by the proportion of total instances in a bucket. This process is repeated for every bucket and the individual scores are summed up to calculate ECE.

2.2 Self-consistency as a measure of confidence

Self-consistency (Wang et al., 2022) is a technique for natural language reasoning that involves using chain-of-thought prompting to generate multiple paths for reasoning. This process aims to select the

most consistent answer by sampling and marginalizing. Here we use a latent variable Z to represent the reasoning chain/programs. Y is the answer that is either extracted in case of CoT or obtained after execution in case of PaL. We marginalize over Z by taking a majority vote over answers. Thus we rely on majority voting over the answers for obtaining confidence estimates for each sample.

K is a hyperparameter that controls the number of generations (referenced in equation 3). The higher the value of K , the better our approximation of the probability of each sample. An overview of this process is shown in Figure 2.

$$P(\hat{Y}_0|Z_0) = \frac{1}{K} \sum_{i=0}^K \mathbb{I}\{\hat{Y}_i = \hat{Y}_0\} \quad (3)$$

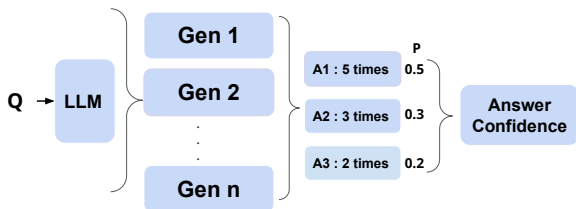


Figure 2: An illustration of obtaining model confidence through majority voting over the answers ($A_1, A_2 \dots A_n$).

Wang et al. (2022) and Xiong et al. (2023a) suggest that self-consistency can be an effective way to elicit confidence from models. Hence, given the lack of per-token log probabilities in closed LMs like gpt-3.5-turbo and text-davinci-003, we adopt self-consistency as a proxy measure for calibration.

2.3 Similarity and Answer Entropy

In addition to empirically evaluating the impact on accuracy and calibration, we conduct a qualitative analysis of the reasoning chains (which can be thought of as the latent variable Z described previously). Here, we observe a consistent pattern: the correct answers corresponding to a question were often associated with similar generations. This observation led us to hypothesize that this phenomenon could be attributed to the fact that there are numerous ways in which solutions can be incorrect, whereas correct solutions tend to exhibit more uniform behaviour (Li et al., 2022). To empirically assess this hypothesis, we employed sentence embeddings generated from the *all-MiniLM-v6* model

to compute the average similarity among the generations which is equivalent to calculating similarity over latent variables Z .

Furthermore, to gain deeper insights into the relationship between similarity in generations and corresponding answers, we also compute the entropy $H(A)$ of the answer space where $P(a_i)$ refers to the probability of the i^{th} answer in K answers obtained by extraction or program execution for a given sample.

$$H(A) = - \sum_{i=1}^K P(a_i) \cdot \log_2 P(a_i) \quad (4)$$

This allowed us to investigate whether the observed similarity in the latent variable space Z leads to a lower entropy within the answer space. These quantitative measures were useful in gaining insights into why specific dimensions yielded more favourable evaluation metrics.

3 Experimental Design

3.1 Models

We compare the calibration and accuracy of two different prompting strategies - CoT and PaL on an equal number of closed-source and open-source models. The open source models used in experimentation are LLaMA2-13B, LLaMA2-70B and the closed-source models are gpt-3.5-turbo, text-davinci-003 (Brown et al., 2020). It should be noted that all models have received some form of supervision from code during pre-training (OpenAI, 2023; Touvron et al., 2023), in addition to being primarily trained on text.

3.2 Hyperparameters

For our experiments, we set temperature (T) as 1.0 and the probability (p) for nucleus sampling (Holtzman et al., 2020) as 1.0. Selecting a temperature of 1.0 enables direct sampling from the model as there is no scaling of probabilities involved, as seen from Equation 5. Here, z_i refers to the logit for the i th token generated and N is the size of the vocabulary.

$$\sigma(z_i) = \frac{e^{\frac{z_i}{T}}}{\sum_{j=0}^N e^{\frac{z_j}{T}}} \quad (5)$$

For each sample in a dataset, we set the number of generations (K) as 10. For each generation, we set the maximum number of tokens (input + output) at 1024.

Dataset	Category	# Samples	Example
GSM8K (Cobbe et al., 2021)	Arithmetic	1319	Q: A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take? A: 3
GSM8K Hard (Gao et al., 2022)	Arithmetic	1319	Q: A robe takes 2287720 bolts of blue fiber and half that much white fiber. How many bolts in total does it take? A: 3431580
Date Understanding (Suzgun et al., 2022a)	Symbolic	360	Q: Yesterday was April 30, 2021. What is the date today in MM/DD/YYYY? A: 05/01/2021
Object Counting (Suzgun et al., 2022a)	Algorithmic	250	Q: I have three couches, a lamp, a stove, a table, a fridge, and a microwave. How many objects do I have? A: 8
Repeat Copy (Suzgun et al., 2022a)	Algorithmic	32	Q: say python twice and data once, and then repeat all of this three times. A: python python data python python data python python data

Table 1: Datasets with their examples and categories.

3.3 Tasks

We examined reasoning tasks encompassing several challenges that include arithmetic, algorithmic, and symbolic reasoning. We use five datasets that cover these different kinds of reasoning tasks. The arithmetic reasoning datasets include *GSM8K* (Cobbe et al., 2021) and *GSM8K Hard* (Gao et al., 2022). The algorithmic reasoning tasks include *Object-Counting* (Suzgun et al., 2022a) and *Repeat-Copy* (Suzgun et al., 2022a). We used *Date-Understanding* as a Symbolic Reasoning Dataset (Suzgun et al., 2022a). Specific information about the datasets used can be found in Table 1.

3.4 Prompt Design

We provide all models with natural language chain-of-thought (CoT) prompts and code-based Program-Aided Language Model (PaL) prompts. For datasets where CoT prompts are available in their original form, we use them as presented in the original paper (Wei et al., 2022). For other datasets, we modify these prompts to suit the specific task while maintaining their original format. For PaL prompts we use and adapt the code-prompts provided in (Gao et al., 2022). The prompts can be seen in Appendix Section A.

4 Results

We investigate two model types: OpenAI models and LLaMA models along with the two different prompting strategies - PAL and CoT.

4.1 Effect of prompting style on Calibration

In this section, we look at the first two *RQs*:

RQ 1: Does one prompting style result in significantly better calibration than the other?

RQ 2: Are the observed calibration trends different across OpenAI models and LLaMA models?

Table 2 shows results for OpenAI models, in which we can see that PAL prompting improves both calibration and accuracy across all datasets. We see approximately 50% relative reduction in calibration error and an average improvement of 18.42% in accuracy. In Figure 3 we show reliability diagrams, an illustration of the bucket values from Equation 2. These provide an illustration of improved calibration, with the reliability curves for PAL prompting consistently aligning closer to the ideal reliability curve as compared to CoT across datasets. While PAL shows a notable gain of 14.83% in accuracy across all datasets for LLaMA models, it shows better calibration in only half of our settings. Overall for both OpenAI models and LLaMA models, we observe that PAL leads to better calibration than CoT for 75% of the settings.

Effect of PAL on calibration controlling for accuracy One reasonable hypothesis is that PAL is mainly improving calibration because it achieves higher accuracy, and more accurate models can be better calibrated. ‘To examine this hypothesis, we conduct statistical analysis using *mixed linear models* (McLean et al., 1991), which allow us to consider the significance of varying the prompting strategy while controlling for accuracy as a confounding factor.

Upon analyzing the results in Table 3, we observe that, when treating the prompting style as a fixed effect, PAL exhibits a negative coefficient of -0.103 (p=0.0) for OpenAI models which is statistically significant with a threshold of p=0.05. This implies that PAL contributes to the reduction in

Name	Score	Model	GSM8K		Object-Counting		Repeat-Copy		Date-Understanding		GSM8K Hard	
			CoT	PaL	CoT	PaL	CoT	PaL	CoT	PaL	CoT	PaL
LLaMA2-70B	ECE (↓)	LLaMA	0.19	0.07	0.17	0.14	0.18	0.23	0.09	0.18	0.07	0.03
	ACC (↑)	LLaMA	59.28	63.91	76.00	92.40	40.62	71.88	66.66	70.18	21.45	40.62
	SIM (↑)	LLaMA	72.20	92.40	94.43	94.72	87.10	90.58	86.87	82.15	92.28	74.32
	ENT (↓)	LLaMA	2.24	1.92	1.00	0.76	1.93	2.00	1.44	1.54	2.85	2.17
LLaMA2-13B	ECE (↓)	LLaMA	0.06	0.08	0.08	0.06	0.11	0.17	0.06	0.05	0.12	0.14
	ACC (↑)	LLaMA	27.0	34.34	56.4	81.6	34.37	53.12	48.24	50.41	6.67	25.55
	SIM (↑)	LLaMA	76.6	93.3	93.2	95.3	89.8	88.6	79.5	84.2	74.0	92.32
	ENT (↓)	LLaMA	2.83	2.49	1.52	0.85	2.43	2.47	2.23	2.06	2.42	3.06
text-davinci-003	ECE (↓)	OpenAI	0.04	0.03	0.29	0.02	0.20	0.06	0.19	0.11	0.15	0.07
	ACC (↑)	OpenAI	65.65	76.49	59.21	98.00	67.23	93.75	60.70	72.35	23.95	71.27
	SIM (↑)	OpenAI	90.5	97.8	99.1	99.8	96.2	98.2	92.4	97.4	89.8	97.9
	ENT (↓)	OpenAI	1.27	0.79	0.36	0.02	1.38	0.44	0.71	0.64	2.31	0.81
gpt-3.5-turbo	ECE (↓)	OpenAI	0.05	0.03	0.38	0.03	0.18	0.16	0.17	0.13	0.13	0.05
	ACC (↑)	OpenAI	84.00	82.40	82.40	97.20	56.25	68.75	61.51	77.23	55.21	62.91
	SIM (↑)	OpenAI	94.40	97.80	99.10	98.60	97.70	97.90	95.3	97.6	90.60	95.40
	ENT (↓)	OpenAI	0.57	0.49	0.59	0.048	1.15	0.35	0.50	0.36	1.65	2.43

Table 2: Comparison of Expected Calibration Error (ECE (↓)), Accuracy (ACC (↑)), Cosine Similarity (SIM (↑)) and Answer Entropy (ENT (↓)) across datasets. The darker blue shade highlights better performing prompting technique.

Model Type	LLaMA models	OpenAI models	Both
Fixed Effect (ECE vs Prompting Style)	PAL : -0.010	PAL : -0.103	PAL : -0.067
p-value	0.961	0.000	0.002

Table 3: Statistical analysis using *mixed-LM*, keeping ECE vs Prompting Style as a fixed effect and accuracy as a random effect.

ECE, and has a positive impact on calibration. On the contrary, for LLaMA models, we did not find that PaL had a statistically significant effect on ECE after controlling for accuracy. Across LLaMA models and OpenAI models, PaL has a statistically significant ($p=0.02$) correlation of -0.067 with ECE, indicating that PaL helps increase calibration on the whole even when controlling for accuracy.

To summarize, we see that PaL prompting has better calibration than CoT prompting ($-RQ1$). While PaL has improved calibration in all settings for OpenAI models, this trend is less consistent for LLaMA models ($-RQ2$).

4.2 Effect of generation diversity on calibration

In this section, we look at the third research question: **RQ 3: Does the consistency of LLM generations affect calibration?**

Qualitative analysis of the generations reveals that PaL generations adhere to a consistent structure that divides the problem-solving process into three distinct parts. This is depicted in Figure 4. In the first part, the model initializes the variables and sets up their initial values required for the calculation. This part is straightforward due to syntactic

constraints and therefore remains largely similar across generations. In the second part, the model generates the required logic by manipulating variables, applying formulas, and utilizing various operations to derive the desired result. Finally, in the third part, the model generates the answer by assigning the calculated value or result to a variable and returning it, which again doesn't vary much across generations. Hence, the diversity of the generation is mostly limited to the second part making code more constrained in its generation space compared to text. Hence we observe a **standardized structure in the code** generated by language models with PaL prompts.

Lower generation diversity and answer entropy observed in prompting strategy with better calibration To quantitatively analyze if code-based generations have lower generation diversity and hence lead to a narrower answer space, we computed aggregated cosine similarity scores for all the generations and entropy over the answer space.

For OpenAI models, we note that the cosine similarity scores with PaL are higher than the corresponding scores for CoT. This observation suggests that, from a semantic perspective code-based generations display a higher degree of similarity. Moreover, the answer entropy for PaL is lower than CoT. This implies that similar generations that cluster together in the semantic space (Li et al., 2022), also converge to the similar solution space. This leads to lower uncertainty in the probability distribution of the answer space and hence lower entropy. From Table 2, we thus can see that PaL

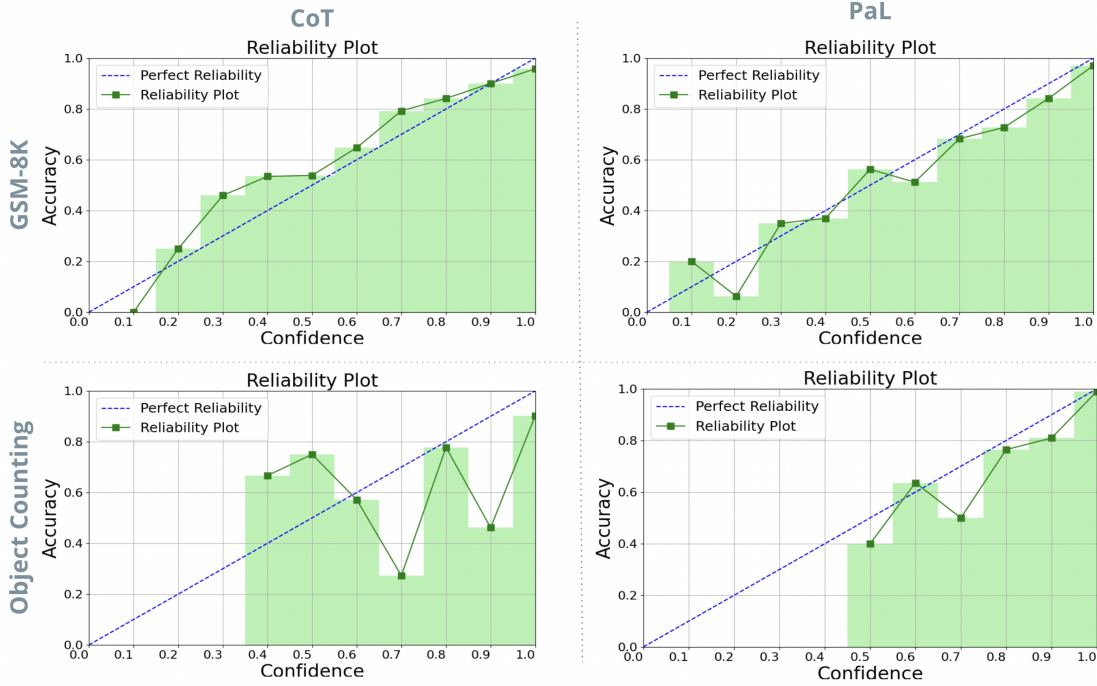


Figure 3: Reliability Plots for various kinds of structured reasoning tasks for the model gpt-3.5-turbo . The x-axis represents confidence and the y-axis represents accuracy.

```
def solution () :
    # Part 1: Initialize
    num_glasses = 16
    first_glass_price = 5
    second_glass_discount = 0.6

    # Part 2: Calculate
    second_glass_price = first_glass_price *
                        second_glass_discount
    pair_price = first_glass_price +
                second_glass_price
    num_pairs = num_glasses // 2
    total_cost = num_pairs * pair_price

    # Part 3: Result Generation
    result = total_cost
    return result
```

Figure 4: Typical output structure with PaL

helps produce similar generations that converge to the same answer space, which is also consistently correct. Hence, achieving better performance and providing more confidence in its predictions.

For LLaMA models, we don't see this trend of PAL having higher generation similarity and lower answer entropy for all datasets. However, for almost all settings for LLaMA models and OpenAI models, the prompting strategy that produces more similar generations and lower answer entropy is also more calibrated.

To summarize, it is evident that lower generation

diversity and lower answer entropy are correlated with higher calibration. (*-RQ3*)

Better calibration observed for PAL when inducing similarity in generations for LLaMA2-70B

We observe that for OpenAI models, PAL is not only more accurate but also more calibrated than CoT. Consequently, we explore whether the reduction in generation diversity, achievable through lower temperatures, can contribute to improved calibration for LLaMA models.

We perform a parameter sweep across temperature values ranging between 0.1 and 0.7 with a step size of 0.2. We show the variation of accuracy, calibration, generation similarity, and answer entropy for two datasets in Figure 5. The plots for the remaining datasets are available in Appendix B, Figure 6. We can see that we obtain better calibration in LLaMA2-70B in both PAL and CoT for temperatures below 1.0. From Table 4 we note that in the majority of runs with $T < 1.0$, PAL is better calibrated than CoT. Optimal performance, considering accuracy and calibration, is achieved at different temperatures for each dataset. For most T values, we note that the similarity scores are higher while corresponding answer entropy values are lower for PAL compared to CoT. This mirrors the pattern observed for OpenAI models.

However, optimal temperature values in our runs

Temp	GSM8K		Object-Counting		Repeat-Copy		Date-Understanding		GSM8K Hard		
	CoT	PaL	CoT	PaL	CoT	PaL	CoT	PaL	CoT	PaL	
0.7	ECE	0.101	0.07	0.06	0.03	0.14	0.12	0.12	0.09	0.18	0.03
	ACC	66.03	67.9	77.6	93.2	53.1	75.0	74.5	76.42	27.14	52.91
	SIM	85.07	97.47	98.53	99.42	93.78	94.81	89.62	96.16	83.28	97.29
	ENT	1.60	1.48	0.55	0.21	1.46	1.35	0.88	0.80	2.43	1.72
0.5	ECE	0.049	0.036	0.103	0.059	0.112	0.075	0.114	0.063	0.139	0.104
	ACC	66.94	67.24	77.23	92.4	59.3	68.75	73.44	77.2	27.7	51.63
	SIM	88.69	98.25	99.17	99.85	97.09	96.81	92.49	97.97	87.65	98.2
	ENT	1.35	1.19	0.39	0.12	1.09	0.99	0.60	0.52	2.18	1.39
0.3	ECE	0.057	0.097	0.140	0.064	0.194	0.113	0.153	0.139	0.230	0.206
	ACC	64.89	63.38	78.8	91.2	53.12	71.87	72.62	76.42	26.16	49.28
	SIM	91.91	98.75	99.51	99.94	97.73	98.27	95.18	99.02	91.14	98.75
	ENT	1.087	0.960	0.238	0.056	0.780	0.504	0.420	0.317	1.866	1.076
0.1	ECE	0.219	0.257	0.188	0.07	0.278	0.156	0.233	0.176	0.418	0.380
	ACC	58.6	58.37	77.2	90.4	53.12	68.75	69.91	78.32	23.5	45.87
	SIM	95.79	99.37	99.82	99.98	99.28	99.64	98.21	99.68	95.31	99.35
	ENT	0.661	0.526	0.085	0.026	0.288	0.173	0.195	0.137	1.179	0.540

Table 4: Results of temperature scaling for one of the LLaMA models - LLaMA2-70B. The darker blue shade highlights better performing prompting technique.

for calibration are either 0.5 or 0.7, while extreme values (0.1, 1.0) yield lower calibration and accuracy performance. We can therefore see that scaling temperatures in the LLaMA models can help us to obtain better calibration for PaL, which already performs better than CoT on these reasoning tasks.

Overall, we see that lower generation diversity and lower answer entropy lead to higher calibration up to a certain point, after which it negatively affects the calibration. (*-RQ3*)

5 Related Work

5.1 Prompting Strategies for Reasoning

Recent developments in language models have introduced various methods to enhance their reasoning abilities. One such method is CoT (Wei et al., 2022) which helps models generate a series of intermediate steps to solve problems. CoT has demonstrated improved performance in tasks involving arithmetic, common sense, and symbolic reasoning. There are approaches such as PaL (Gao et al., 2022) and Program-of-thoughts (PoT) (Chen et al., 2022) which go a step further by generating programs as intermediate steps and using an interpreter to process them. Code as a medium of reasoning has shown considerable promise evidenced by better performance over chain-of-thought style prompting strategies, in several recent studies (Madaan et al., 2022; Gao et al., 2022; Lyu et al., 2023; Zhang et al., 2023a,b). Different from these works, our main goal in this paper is to understand the effect

of code prompts on calibration.

5.2 Calibration in Language Models

Calibration has been extensively studied in structured prediction problems, such as named entity recognition and part of speech tagging (Jagannatha and Yu, 2020), as well as in natural language understanding tasks, like question answering and text classification (Kamath et al., 2020; Kong et al., 2020; Desai and Durrett, 2020). More recently, studies have directed their attention to calibrating language models when used as generators (Jiang et al., 2021; Zhao et al., 2021). Additionally, the study by Kadavath et al. (2022) explored the likelihood of a model knowing the answer before proposing a response. However, all of these approaches typically rely on access to the model’s logits.

In contrast, the work by (Tian et al., 2023) investigates verbalized probability estimates to assess the calibration of large language models without needing access to logits. This involves the practice of querying the model about its confidence in the answers it generates. Furthermore, (Xiong et al., 2023b) introduced self-consistency-based methods for calibration, demonstrating their superior performance compared to verbalized methods. In our research, we adopt self-consistency as the method of choice for measuring calibration.

6 Conclusion

In this study, we explore the impact of two distinct prompting styles, namely PaL and CoT, on the

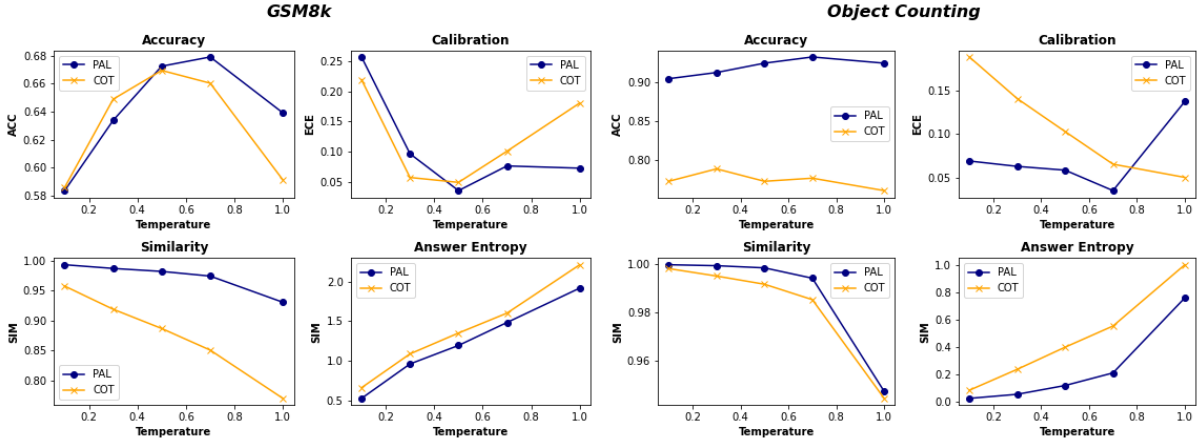


Figure 5: Trends seen in temperature scaling for the model LLaMA2-70B. Across datasets, the accuracy and calibration improve upon lower the temperature up to a certain extent. This is in line with having lower generation similarity and lower answer entropy. The optimal temperatures seen are 0.5 and 0.7 across datasets. For other datasets, refer Appendix, Figure 6.

calibration of OpenAI models and LLaMA models. Our investigation spans 5 reasoning datasets, employing self-consistency as the methodology for eliciting calibration. We analyze four different metrics - calibration (ECE) , accuracy (ACC) , average similarity in generations (SIM) , and answer entropy (ENT) . We summarize our findings as follows:

- **RQ 1:** *Does one prompting style result in significantly better calibration than the other?* Empirical results show that PAL generally has higher calibration and accuracy for 82.5% of the cases across OpenAI and LLaMA models for a varied range of temperatures.
- **RQ 2:** *Are the observed calibration trends different across OpenAI models and LLaMA models?* We observed that OpenAI models are in general better calibrated for the reasoning tasks with up to 19% improvement in ECE score.
- **RQ 3:** *Does the consistency of LLM generations affect performance?* PAL prompting shows a general trend of having greater similarity in the generation of up to 7% over text, which we hypothesize could be due to the inherent structure present in the code. We see that greater generation similarity is accompanied by lower answer entropy and lower ECE. However, temperature scaling experiments reveal that reducing generation diversity helps improve calibration only up to certain temperature values – the calibration is affected

negatively for lower temperatures such as 0.1 and 0.3.

We hope that this study will serve as a catalyst for additional research aimed at holistically evaluating and gaining deeper insights into the role of prompts in various task domains across other dimensions in addition to accuracy.

7 Acknowledgments

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A Prompts

The following sections display one example of the few-shot prompts used for each dataset across prompting styles.

A.1 PAL Prompts

A.1.1 GSM8K/GSM8K-Hard

```
def solution () :  
    """Olivia has $23. She bought five bagels for $3 each. How much money does she have left?"""  
    money_initial = 23  
    bagels = 5  
    bagel_cost = 3  
    money_spent = bagels * bagel_cost  
    money_left = money_initial - money_spent  
    result = money_left  
    return result
```

A.1.2 Object Counting

```
# Q: I have a chair, two potatoes, a cauliflower, a lettuce head, two tables, a cabbage, two  
↪ onions, and three fridges. How many vegetables do I have?  
...  
def solution () :  
    # note: I'm not counting the chair, tables, or fridges  
    vegetables_to_count = {'potato': 2, 'cauliflower': 1, 'lettuce head': 1, 'cabbage':  
↪ 1, 'onion': 2}  
    return sum (vegetables_to_count.values () )  
...
```

A.1.3 Date Understanding

```
# Q: 2015 is coming in 36 hours. What is the date one week from today in MM/DD/YYYY?  
# If 2015 is coming in 36 hours, then today is 36 hours before.  
today = datetime (2015, 1, 1) - relativedelta (hours=36)  
# One week from today,  
one_week_from_today = today + relativedelta (weeks=1)  
# The answer formatted with %m/%d/%Y is  
one_week_from_today.strftime ('%m/%d/%Y')
```

A.1.4 Repeat Copy

```
# Q: Repeat the word duck four times, but halfway through also say quack  
...  
def solution () :  
    result = []  
    for i in range (1, 5) :  
        result.append ("duck")  
        if i == 2:  
            result.append ("quack")  
    return " ".join (result)  
...
```

A.2 CoT Prompts

A.2.1 GSM8K/GSM8K-Hard

Q: There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

A: There are 15 trees originally. Then there were 21 trees after some more were planted. So there must have been $21 - 15 = 6$. The answer is 6.

A.2.2 Object Counting

Q: I have a chair, two potatoes, a cauliflower, a lettuce head, two tables, a cabbage, two onions, and three fridges. How many vegetables do I have?

A: Chair, tables and fridges are not vegetables, so we are not counting them. Two potatoes, cauliflower, lettuce head, cabbage and two onions are vegetables, so they will be counted. The total number of vegetables are $2 + 1 + 1 + 1 + 2$. The answer is: 7

A.2.3 Date Understanding

Q: 2015 is coming in 36 hours. What is the date one week from today in MM/DD/YYYY?

A: If 2015 is coming in 36 hours, then it is coming in 2 days. 2 days before 01/01/2015 is 12/30/2014, so today is 12/30/2014. So one week from today will be 01/05/2015. So the answer is 01/05/2015.

A.2.4 Repeat Copy

Q: Repeat the word duck four times, but halfway through also say quack

A: On repeating the word duck four times we get: duck duck duck duck. Halfway through if we say quack, we have to say quack in between the 2nd word and the 3rd word. The answer is: duck duck quack duck duck

B Temperature Scaling Experiments - Line Plots

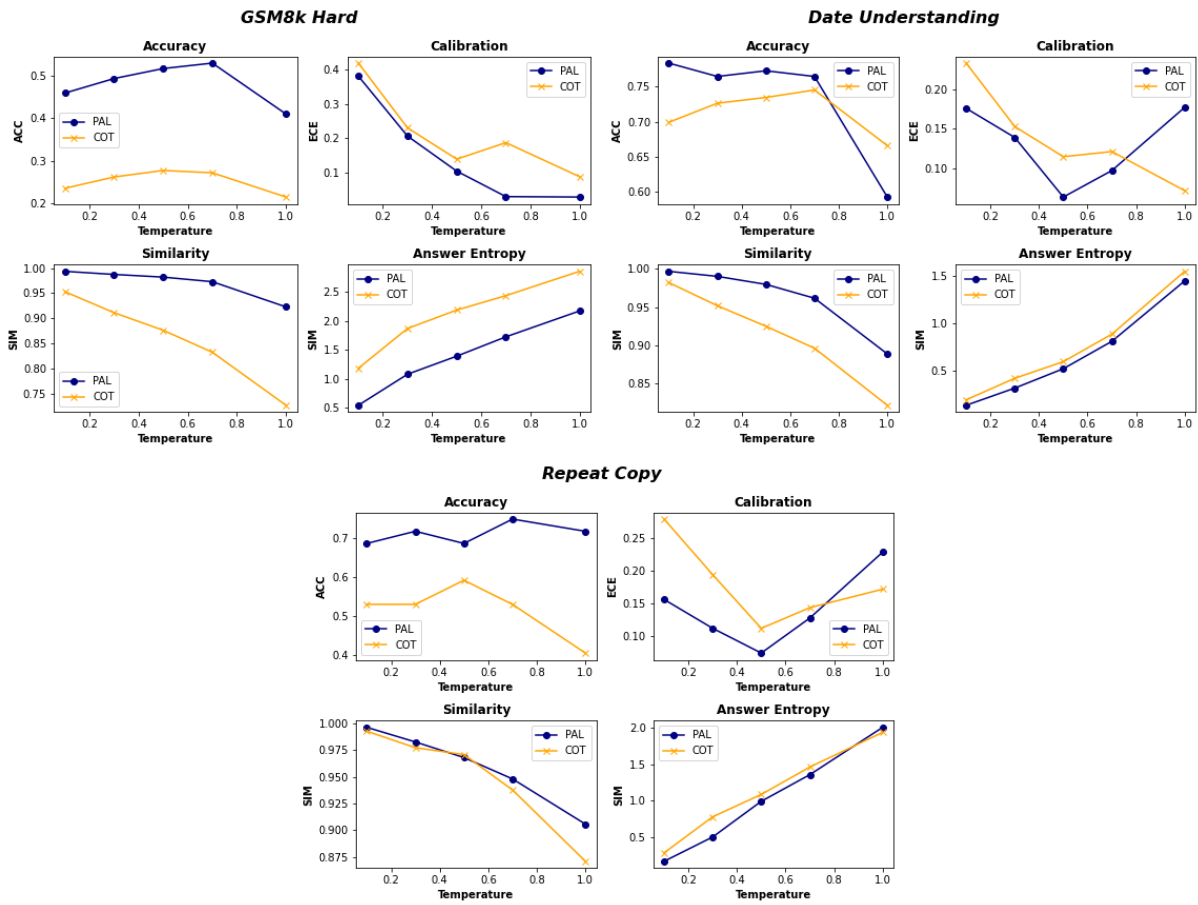


Figure 6: Trends seen in temperature scaling for GSM8K Hard, Date Understanding and Repeat Copy